Co-operative inversion of geoelectrical data sets acquired from different electrode arrays

Duy Thong Kieu*  
Hanoi University of Mining and Geology  
18 Pho Vien, Duc Thang, Bac Tu Liem  
Hanoi, Vietnam  
kieuduythong@humg.edu.vn

Thi Chinh Truong  
Hanoi University of Mining and Geology  
18 Pho Vien, Duc Thang, Bac Tu Liem  
Hanoi, Vietnam  
truongthichinh@humg.edu.vn

SUMMARY
Direct current resistivity (DCR) method is one of the most commonly applied geophysical exploration methods. The development of data acquisition techniques enables the acquisition of multiple data sets of various electrode arrays with a little extra measurement time in comparison with the time that needs to install the system. Accordingly, the data processor is required to utilise as much as possible useful information to build a more reliable geoelectrical model. This study aims to test using the co-operative inversion process to the multiple data sets of various electrode configurations. We use a synthetic model with the most common electrode arrays: Wenner-Schlumberger (WS), Dipole-Dipole (DD), Pole-Dipole (PD) and Pole-Pole to investigate the possibility of the co-operative inversion schemes. The results show that the co-operative inversion of the combined data sets is better than the inversion of the individual ones. The order of inversion for each data set can produce different results. Fuzzy c-means constraint may assist the inversion to produce better results.

Key words: direct current, electrode array, co-operative inversion, joint inversion, fuzzy clustering.

INTRODUCTION
Direct current resistivity (DCR) method is one of the most commonly applied geophysical methods. In recent years, this method has obtained significant developments including instrumentation, field survey design and data inversion techniques (Loke et al., 2013). The improvement of the acquisition techniques that allow the collection of multiple data sets from different electrode configurations with slightly extra time. Each array type has a different resolution, for instance, DD array is better than WS array for lateral resolution while WS is of higher vertical resolution than DD (Novák et al., 2009). Separate inversion of individual data sets for each array type may produce different geoelectrical models due to the non-unique solutions of the inversion. The optimal solution, in this case, is to utilise all available data in an inversion scheme to build a potentially more reliable model. The question is how to put all the data in the inversion approach.

Athanasiou et al. (2007) proposed the combined weighted inversion approach. However, it is difficult to conclude whether the weighted inversion is better than the un-weighted inversion (Candansayar, 2008). Additionally, a critical issue for the weighted inversion approach is how to define the weighting factor. Regarding this aspect, the co-operative inversion may be better. However, the order of data sets in the co-operative inversion process may produce different results. For example, inversion of data sets of WS and DD can produce two models depending on which data set WS or DD is inverted first. This research demonstrates the use of the co-operative inversion of multiple data sets from multiple electrode configurations.

Figure 1. The four commonly used electrode arrays. C1 and C2 are a current pair, P1 and P2 are a potential pair.

Table 1. The depth of investigation (z*) and vertical solution (∆z) of the three arrays (After Novák et al., 2009). R is array length.

<table>
<thead>
<tr>
<th>Array name</th>
<th>z*/R</th>
<th>Vertical resolution (R/∆z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>0.125</td>
<td>4.0</td>
</tr>
<tr>
<td>DD</td>
<td>0.19</td>
<td>3.0</td>
</tr>
<tr>
<td>PD</td>
<td>0.235</td>
<td>2.2</td>
</tr>
<tr>
<td>PP</td>
<td>0.355</td>
<td>1.2</td>
</tr>
</tbody>
</table>

METHOD AND RESULTS

Electrode Arrays
In this study, we test four electrode types WS, DD, PD and PP (Figure 1). Each array type has a different resolution and depth of investigation (Table 1). The purpose of our tests is to find how to exploit complementary information from each array type to build a more reliable model. For instance, a
combination of data of high vertical resolution (WS) of high lateral resolution (DD and PD) may produce better results.

**Inversion Methodology**

**Separate inversion**

The critical issues of inversion are to deal with non-unique solutions, to solve this problem extra information is needed to constrain the inversion. A commonly used method is smooth constraint (de Groot-Hedlin and Constable, 1990). However, both sharp and smooth geological boundaries exist in nature, thus, smoothness solutions may lead to misinterpretations of geological information. We propose to use an extra term of constraint, fuzzy c-means (FCM) clustering techniques (Sun and Li, 2011). The idea of using clustering is to base on the fact that the typical subsurface structure consists of geological units of nearly uniform conductivity (clusters). Thus, the model construction is more reliable if the grouping criteria are added to constrain the inversion process (Kieu et al., 2017).

In this study, the objective function of the inversion algorithm is minimized as follows (Kieu et al., 2017; Sun and Li, 2011):

\[ \Phi = \Phi_d + \beta \Phi_m + \gamma \Phi_{FCM} \]  

where \( \Phi_d \) is the difference between observed and synthetic data from the inverted models, \( \Phi_m \) represents the smooth constraint and \( \Phi_{FCM} \) is the FCM objective function (Equation 2). The regularization parameters \( \beta \) and \( \gamma \) balance between misfit, model structure and FCM constraint terms.

\[ \Phi_{FCM} = \sum_{j=1}^{N} \sum_{k=1}^{C} u_{jk}^q \| z_j - v_k \|^2 \]  

where \( q \) is the fuzziness parameter, \( q > 1 \), in this study \( q \) is set to be 2 (Bezdek et al., 1984), \( u_{jk} \) is the membership degree of sample \( j \)th belong to the \( k \)th cluster, with the constraint \( \sum_{k=1}^{C} u_{jk} = 1 \), \( v_k \) is the centre value of the \( k \)th cluster, \( C \) is cluster number, in our study this number is predefined. \( N \) is number of samples.

The error of inversion is given by

\[ \text{Error} = 100\% \sqrt{\frac{\sum_i \Delta d_i^2}{\sigma^2}} \]  

where \( \Delta d \) is the difference between measured and synthetic data, \( \sigma \) is the standard deviation of the data.

**Co-operative inversion**

We propose to apply co-operative inversion to the data sets. The advantage of this approach compared with joint inversion (Athanasiou et al., 2007) is that does not require scale the data sets of different electrode arrays. However, the drawbacks of the co-operative inversion are the error of previous inversion results may accumulate to the next inversion round, and the resolution of the co-operative inversion depends strongly on the resolution of the last inversion data set. In this study, we tested the two schemes of co-operative inversion (Figure 2).

In Scheme B of the co-operative inversion, we use FCM clustering to analyse the output of the previous process before putting in the next inversion (Kieu et al., 2018). The purpose of this step is to reduce the errors (artefacts) of the previous inverted model. The resistivity of input is defined by FCM clustering. After the first inversion, we obtain the resistivity, these values is put in FCM clustering Equation 2. The output of FCM clustering is membership degree and centre values, we utilise these results to calculate new resistivity as the following equation:

\[ s_j = \frac{\sum_{k=1}^{C} u_{jk}^q v_k}{\sum_{k=1}^{C} u_{jk}^q} \]  

Figure 2. Co-operative inversion workflows. In Scheme A, the final inverted results of data 1 are directly put in the inversion process for data 2. Scheme B is the same as A, but the output of the previous inversion is analysed by FCM before being put in the next inversion. Noting that this workflow can expand to more than two data sets.

Figure 3. A geoelectrical model comprises of three layers, L1, L2 and L3 and three local objects, O1, O2 and O3. The synthetic data is generated by using the software Res2dmod (M.H. Loke) for the four array types and added 5% Gaussian random noise.
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**Inversion approaches**

We modified the 2D DCR inversion code from (Akca, 2016) whilst retaining the original forward solution. An inversion is performed with synthetic data from the model (Figure 3). The acronyms of array types are set in order of inversion, for example, the inversion of WS and DD datasets, WS+DD or DD+WS present that the WS or DD data is run first respectively.

**Inversion Results**

Figure 4 presents the results of separate inversion of data of individual arrays. Three results of DD, PD and WS data sets can detect two local objects O1 and O2, but the inversion of PP cannot recover these objects. The object O3 can be seen in the inversion of DD and PD data sets, but it is invisible in the model of WS data. The result of WS shows more clearly the difference of resistivity between media L2 and L3 than all other data sets. The result of PD data shows marginally better than DD in the deeper part of the section, particularly in the bottom left corner, due to the fact that its depth of investigation is larger than DD array’s depth of investigation (Table 1). Overall, the inversion of DD and PP data stands for the best and worst cases.

![Figure 4. Comparison between inversion results of the electrode array datasets. The inversion of DD data shows the best result. In contrast, the inversion of PP data is the worst. It is not able to recover the three object O1, O2 and O3 and the difference between L2 and L3, only boundary between L1 and L2 can be seen in the inverted model.](image)

Figure 5. Comparison of errors between different inversion schemes. In general, errors of separate inversion are smaller than those of joint inversion. The inversion of PP data produces the largest error. The errors of co-operative inversion schemes are smaller than those of separate and joint inversion because of better initial models, the initial model of co-operative inversion (Scheme A, Figure 2) is the result of the previous inversion. The order of inversion produces different errors.

![Figure 5. Comparison of errors between different inversion schemes.](image)

Figure 6. Histogram of the inverted resistivity from different schemes of inversions, separate inversion (DD), and co-operative inversion (Scheme B, Figure 2) with different orders PD+DD and DD+PD. The order of PD+DD demonstrates the best result because it shows clearly four clusters, even though the cluster values are higher than the true model values.

![Figure 6. Histogram of the inverted resistivity from different schemes of inversions](image)

The inversions of combination data sets are shown in Figure 7 and Figure 8. The combination of PD and DD shows the best results. This is also consistent with the work of Candansayar (2008). Regarding errors of the inversions (Figure 5), errors of...
all inversions are gradually smaller with iterations, showing that the inversion program is qualified. The errors of joint inversions are higher than those of separate inversions.

The results of joint inversion and co-operative inversion (Scheme A) show not much improvement compared with the results of separate inversion (Figure 4 and 7). The inversion results are improved with the assistance from FCM analysis (Scheme B). Figure 6 shows that the Scheme B of co-operative inversion shows the best results.

CONCLUSIONS

We investigate the use of co-operative inversion approach to run inversion of multiple data sets from multiple electrode arrays. The best combination of the electrodes types is between Dipole-Dipole and Pole-Dipole arrays. The FCM analysis can significantly improve the inversion results.

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REFERENCES

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Figure 7. Comparison between results of joint and co-operative inversion approaches (Scheme A, Figure 2). Noting that the order of array types is set in sequence, for example, the inversion of WS and DD data sets, WS+DD presents the inverted result of WS is run first, vice versa. In general, the combinations of PD and DD produces the best results, while the combination of WS and PP does not improve inverted results compared to the separation inversions.
Figure 8. Comparison results of co-operative inversion Scheme B (Figure 2). Noting that the order of array types are set in sequence, for example, the inversion of WS and DD data sets, WS+DD presents the inverted result with WS being run first, vice versa. The combination of data sets shows improvement in comparison with separate inversion.