Numerical simulation of the dead-volume effect in seismic-frequency measurements of elastic properties of fluid-saturated rocks

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SUMMARY

Low-frequency measurements of the elastic properties rocks based on the forced-oscillation method is an emerging approach for calibration of well log and seismic measurements. The dispersive properties observed in the low-frequency experiments are usually dependent on the boundary conditions of a tested rock sample, whether the sample surface is properly sealed, or surrounded by rigid pore-fluid containers connected with the pore space of the rock, so-called "dead volumes". By treating the dead volume as a specific porous material, we analysed the impact of the dead volumes on the strains in the sample via numerical simulation of the forced oscillation tests, our simulation is based on the models incorporating the properties and physical characteristics of the laboratory samples. The numerical results are in good agreement with the analytical solutions.

Key words: Dead volume, attenuation, bulk modulus

INTRODUCTION

Low-frequency measurements of the elastic properties rocks based on the forced-oscillation method is an emerging approach for calibration of well log and seismic measurements. For fluid-saturated rocks, low-frequency apparatuses aim to measure modulus dispersion and attenuation under confined conditions, where there is no fluid flow through the surface of the tested sample. However, there are unavoidable fluid containers in all experimental low-frequency setups (Pimienta et al. 2016), which are formed by the fluid lines located between the sample ends and the nearby valves. These containers are called 'dead volumes'. The dead volume can be accountable for the pore-fluid flow even under conditions of full saturation and closed pore-fluid lines, and, thus can effect observed attenuation and bulk modulus dispersion.

The effect of the dead volume has been explored using three approaches: laboratory tests, analytical solutions and numerical simulation. Pimienta et al. (2016) proposed a 1-D poroelastic model, which was derived from solving the pore pressure diffusion equation (Zimmerman 2000) with different sets of boundary conditions. Pimienta et al. (2016) demonstrated that the predicted bulk modulus dependence on frequency is consistent with the results of measurements conducted on Fontainebleau sandstone. Using the low-frequency apparatus described in detail in Mikhaltsevitch et al. (2014), Mikhaltsevitch et al. (2019) explored the effect of boundary conditions caused by the dead volume and showed the dependence of the measured bulk modulus of a liquid-saturated rock on the size of the dead volume. Chapman and Quintal (2018) used numerical simulation based on the Biot quasistatic equations for poroelastic media to model the forced oscillation tests on models representative of laboratory samples and showed the effect of local flow on the attenuation and dispersion of the P-wave modulus.

When investigating the effects of pore-fluid movements or comparing different measurements in rocks, one needs a numerical or analytical approach which incorporates boundary conditions and a measurement method. However, the numerical solution by Chapman and Quintal (2018) does not consider the effect of the dead volume, while the method proposed by Pimienta et al. (2016) is limited to the 1-D model. In this study, we present the results of the numerical simulation of the dead-volume effect on the elastic moduli dispersion at seismic frequencies. Our simulation is based on the Biot quasistatic equations. The found results are in good agreement with analytical solutions obtained both mesoscopic and microscopic scales.

METHOD

The displacement–pressure formulation of the Biot consolidation equations can be presented as (Quintal et al., 2011)

$$-\left(\nabla\cdot\boldsymbol{\sigma}\right)=0, \tag{1a}$$

$$\nabla\cdot\left(-\frac{\kappa}{\eta}\nabla\mathbf{p}\right)+\left(1-\frac{K_m}{K_s}\right)\cdot\dot{\mathbf{u}}+\frac{\dot{\mathbf{p}}}{M}=0\,. \tag{1b}$$

where the dot on the top of variables indicates the partial time derivative of the first order and the symbol ∇ is the del operator. The material properties κ and η are permeability and viscosity, respectively. K_s and K_m denote the bulk moduli of the solid grains and dry matrix, respectively, M is the fluid storage coefficient, $\mathbf{p}$ is the pore fluid pressure, and $\mathbf{u}$ is the displacement vector. In the porous medium, when a force is applied to a rock, both the rock matrix and fluids in the pores will bear the force. The amount of force borne by fluids corresponds to the pore pressure. In the 3D isotropic fractured porous media

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu\varepsilon_{ij} - \left(1 - \frac{K_m}{K_s}\right)\mathbf{p}\delta_{ij}, \tag{2}$$

where

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad i, j = 1,2,3 \tag{3}$$

$$e = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}, \tag{4}$$

where ε_{ij} is the strain tensor, e is the cubical dilatation, and δ_{ij} is the Kronecker delta, λ and μ are the Lamé parameters. $\sigma \equiv (\sigma_{ij})$ is the total stress tensor.

The detailed information about the numerical test can be found in Quintal et al. (2011).

Table 1 Physical properties of the solid frame

Property	Porous medium	Dead Volume
Grain		
Bulk modulus K_s	37.0 (GPa)	37.0 (GPa)
Density ρ_s	2.65 (g.cm−3)	2.65 (g.cm−3)
Frame		
Bulk modulus K_m	14 (GPa)	25 (Pa)
Shear modulus μ	31 (GPa)	10 (Pa)
Porosity φ	0.07	0.99
Permeability κ	$4 \cdot 10^{-15}$ (m^2)	Eq.(5)

Table 2 Physical properties of fluid (Pimienta et al. 2016)

Property	Brine
Density ρ	1.09 (g.cm−3)
Bulk	2.25 (GPa)
Viscosity η	0.89e-3 (Pa.s)

In equations (1) – (4), the same parameters describe the behaviour of the porous medium and the fluid in the dead volumes (Figure 1). To ensure the dead volume is filled with water the porosity in the dead volumes is set as 0.99 (Table 2) while the bulk modulus and shear modulus are set to be much smaller than the moduli of the sample (Table 2). The advantage of this approach is that it yields a natural coupling between solid and fluid displacements at the subdomain boundaries. This makes it unnecessary to explicitly impose boundary conditions between fluid and solid domains (Quintal et al. 2016)

Because the direction of the fluid flow in the dead volume is predominantly parallel to their walls, the permeability within the dead volume is estimated using an analytical solution of Navier-Stokes equations for laminar flow between two smooth walls separated by a uniform aperture *h* of the dead volume (Jaeger, Cook, and Zimmerman 2007).

$$\kappa = \frac{h^2}{12} \; . \tag{5}$$

Our numerical strategy is based on solving the same equations (Equations 1 and 2) throughout the entire computational domains consisting of the sample and dead volume, however, the attenuation and phase velocity are obtained solely on the sample domain.

NUMERICAL RESULTS

The model (Figure 1) is composed of two parts: the porous medium and two dead volumes. The medium has a length of 8 cm and a diameter of 7.6 cm, while the dead volume has a radius of 9 mm and a volume of 3.4 ml for the two of dead volume, respectively.

A vertical displacement is applied on the top of the sample (excluding the connection betwee Low-frequency measurements of the elastic properties rocks based on the forced-oscillation method is an emerging approach for calibration of well log and seismic measurements. n the sample and dead volume), while the displacement in the Z-direction at the bottom (excluding the connection between the sample and dead volume), and the X-direction at the left and right boundaries are considered zero. No-flux boundary conditions are applied on the edges of the samples and the dead volumes simultaneously.

After the numerical simulation, we average the vertical components of the stress and strain fields either over the complete sample length, constituting the global response, or over a slice of 0.5 cm thickness, which is accountable for the local response. In the analytical solution (AS), the local response is calculated at a single point based on the solution proposed by Pimienta et al. (2016).

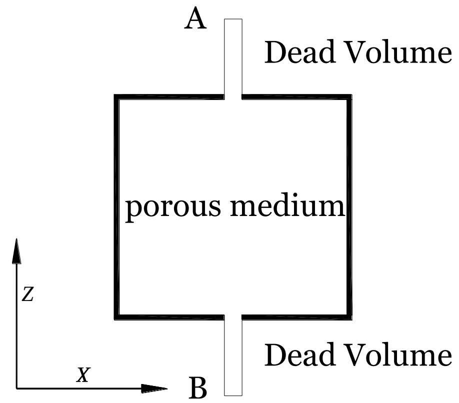

Figure 1. The model of the porous medium with dead volumes

Numerical result shows that, considering the effect of the dead volume, the numerical method could also provide a good fit with that of the attenuation and bulk moduli.

According to Gassmann's equations, the high frequency-limits of bulk modulus is 14 GPa, while the high frequency limit of bulk modulus is 22.4 GPa. And numerical results (blue squares) in Figure 2b coincide well with these two limits.

However, the local response can vary greatly from the global ones due to the fluid flow between the sample and dead volume. Indeed, the local responses also show more variable frequency dependence than the bulk response, corresponding with the behaviour of the attenuation curves, but they converge to the low- and high-frequency limits of the bulk measurement, as observed already for the case shown in Figure 2.

CONCLUSIONS

Based on the quasi-static Biot equation of consolidation, we investigated the influence of the dead volume on the low-frequency measurements of the elastic moduli of the sandstones fully saturated with water. In our analysis, the fluid in the dead volume was treated as a poroelastic material. The advantage of this approach is that it yields a natural coupling between solid and fluid displacements at the subdomain boundaries.

Our numerical results are in good agreement with the analytical solution. The bulk modulus exhibits strong dispersion at frequencies above 10 Hz. However, the effective bulk modulus differs in the local and global responses. The reason may lie in the fluid interplay between porous medium and dead volumes. In the future, this numerical method can be modified to provide a benchmark for the laboratory test.

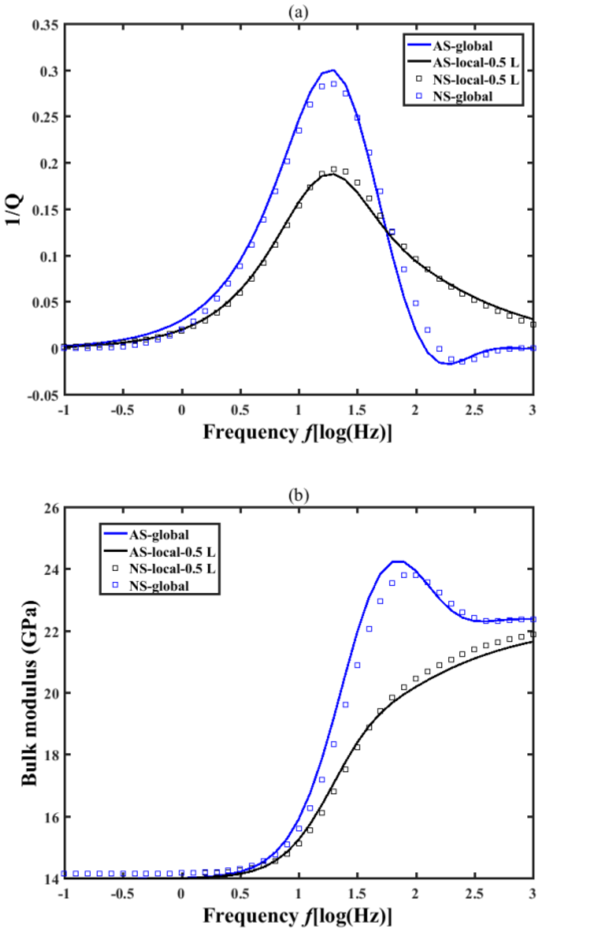

Figure 2. The attenuation (a) and the real part of Young's modulus (b) obtained from the numerical (NS) and analytical (AS) solutions, considering the global and local responses at 50% of the sample length (L).

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