

3-D DC resistivity modeling and inversion using multi-resolution framework

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SUMMARY

We discuss the implementation of multi-resolution framework to 3-D Direct Current (DC) problem. Commonly used staggered (SG) grid fixes the horizontal grid resolution for all depths. Thus, employing the fine horizontal resolution may lead to an over-discretised forward problem, subsequently affecting the performance of the inversion. We implemented a novel multi-resolution (MR) grid approach to the 3-D DC modeling and inversion problem, which allows adjustment of the horizontal resolution with depth. By using finer resolution for the near-surface regions, MR grid can ensure the modeling accuracy and describe the shallow features in the inversion model as well. The ability to use relatively coarser horizontal resolution for the deeper regions reduces the computation costs compare to the SG grid modeling. As a result, modeling and inversion can be accelerated several times by solving a smaller problem. Our grid resembles non-conformal rectangular grid, which commonly used in finite-elements modelling.

Key words: Direct Current Resistivity method; multi-resolution grid; modeling; inversion;

INTRODUCTION

Modeling and inversion algorithms are the necessity for interpretation of geophysical data. Since the 3-D modeling and inversion are computationally expensive, there are many ongoing efforts being made to improve their efficiency.

The 3-D Direct Current Resistivity (DCR) method usually involves measurements with dense electrodes array (Loke et al. (2013)). Therefore, modeling grid needs finer discretization around the current electrodes and shallow anomalies to ensure the modeling accuracy. With increased depth, the variations of the simulated electric potential tend to be smoother and could be matched by the relatively coarser discretization. As the DCR data are gradually losing its exploration sensitivity with depth, deeper parts of the inversion model also have lower demand of the grid discretization.

The standard staggered (SG) grid, uses the rectangular cells for model discretization and fixes the horizontal resolution to all depth. This leads to an over-discretization of the deeper parts, and to excessive increase in computational resources.

In our work, we present a multi-resolution (MR) grid approach, which allows adjusting the horizontal resolution of the grid with depths. MR grid was initially developed to solve the electromagnetic forward problem (Cherevatova et al. (2018)). Here, we adopted it to the 3-D DCR modeling.

METHOD

We utilize the finite-difference method in the secondary field approach (Lowry (1989)). There, the simulated electric potential ϕ is derived from the system of equations:

$$[\mathbf{G}^\dagger \text{diag}(\mathbf{W}\sigma) \mathbf{G}] \phi = \mathbf{J},$$

where \mathbf{G}^\dagger and \mathbf{G} are the discrete divergence and gradient operators, respectively; \mathbf{W} is an averaging operator, which maps conductivity σ to grid edges; \mathbf{J} defines the source term. For inversion, we use the conjugate-gradient method (Mackie and Madden (1993)).

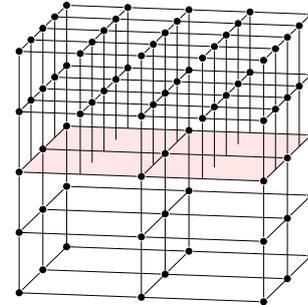


Figure 1. The illustration of the MR grid with parameters $[C_s^{(i)}, N_z^{(i)}]_{i=1,2} = (0, 2; 1, 2)$ derived from a $4 \times 4 \times 4$ SG grid.

MR grid can be represented as a vertical stack of several sub-grids. Each sub-grid is regarded as a standard SG grid with different horizontal resolution. The horizontal resolution of each sub-grid is determined by the following rule:

$$N_x^{(i)} = \frac{N_x^f}{2^{Cs^{(i)}}}, N_y^{(i)} = \frac{N_y^f}{2^{Cs^{(i)}}}.$$

Thus, the MR grid is only controlled by the fundamental SG grid, which uses the finest horizontal resolution: $N_x^f \times N_y^f$, and parameters: $[C_s^{(i)}, N_z^{(i)}]_{i=1, N_{sg}}$, where $C_s^{(i)}$ is the coarseness parameter and $N_z^{(i)}$ is the vertical discretization of the i^{th} sub-grid. Figure 1 shows an example of simple MR geometry.

In order to implement the DCR case on MR grid, the forward modeling operators (\mathbf{G}^\dagger , \mathbf{G} , and \mathbf{W}) and the regularization term in inversion need to be redefined. We employed the Coarse Active (CA) approach (Cherevatova et al. (2018)) to achieve the above requirements (Jungyu Gao et al., 2019).

Synthetic example

Here, we present a simple block model to demonstrate the accuracy and efficiency of MR grid compare to SG grid. A conductive block of $10 \Omega m$ resistivity is embedded into $100 \Omega m$ background. The block size is $60 m \times 60 m \times 30 m$ and it is located at a depth of $30 m$. Three $200 m$ dipole-dipole profiles simulate the measured data with the geometric factors: $\alpha = 20 m$ and $n = 1$ to 8 .

The initial SG grid resolution is $88 \times 64 \times 40$, the central part is discretized with $5 m$ uniform cells. Such discretization results in 219240 nodes and 225280 cells.

The MR grid is derived from the SG grid with the following parameters: $[Cs^{(i)}, Nz^{(i)}]_{i=1,3} = (0,16; 1,14; 2,10)$, and shown in Figure 2. In comparison with SG grid, the MR grid reduces the number of nodes and cells to 109508 and 110944, ca. 50% of SG grid.

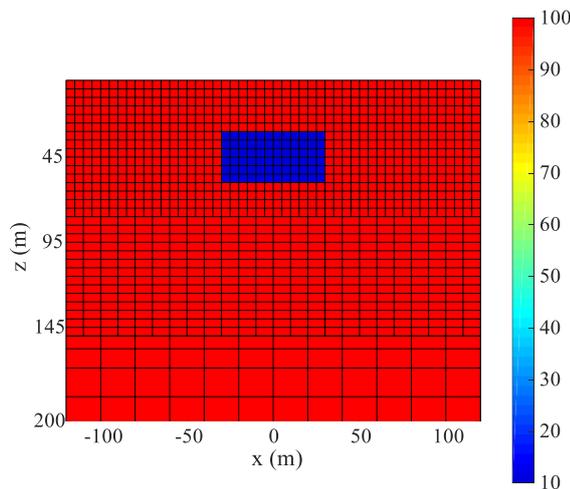


Figure 2. A sketch of the MR grid discretization for a considered block model.

The relative difference ($|\rho_a^{MR} - \rho_a^{SG}| / \rho_a^{SG} \times 100\%$) in the modeling accuracy of both MR and SG cases do not exceed 0.05%. However, the MR result is obtained twice faster than SG. The averaged MR modeling time is 25 s, and the SG – 50 s.

The SG grid modeling result, combined with 5% random noise, was taken as the synthetic observed data for the SG and MR inversions. Both of inversions started with $100 \Omega m$ homogenous half-space, and same starting RMSD of 2.5. The inversions achieved stopping criteria of 1.0 after three iterations with nearly the same RMSD of 0.950 (SG) and 0.945 (MR). In addition, both of their final inversion results are nearly the same, thus, only the MR inversion result is shown in Figure 3.

Comparing the running time of the inversions, MR case is more efficient. It takes 585 s which is only 50 % of the SG inversion time (1185 s).

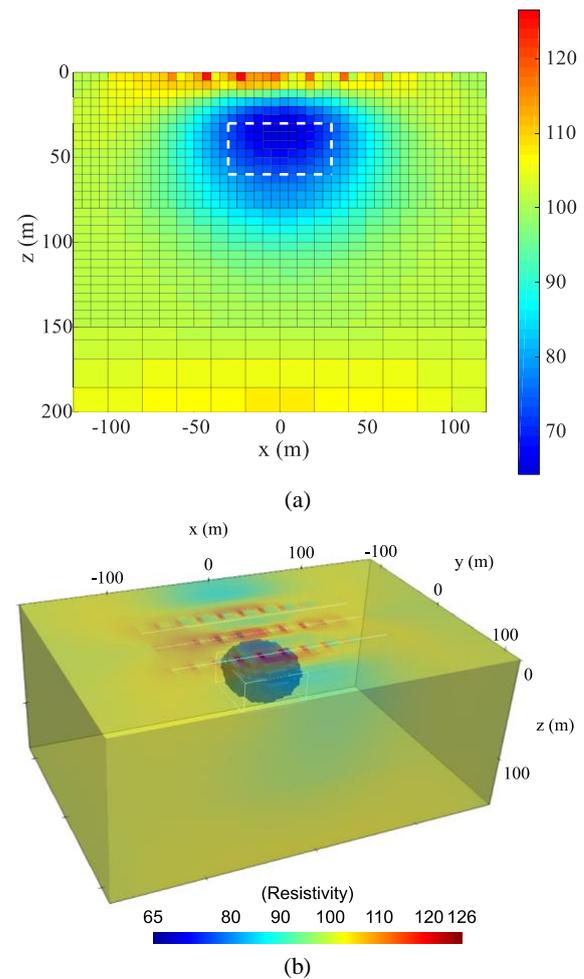


Figure 3. Inversion model based on the MR grid approach.

CONCLUSIONS

We implemented the multi-resolution grid approach to 3-D DCR problem. Considered example shows, that the multi-resolution grid not only maintains the modeling accuracy but also improves on efficiency of the modeling and inversion, as compared to conventional staggered grid approach. The acceleration ratio is nearly linear in proportion to the ratios of amount of the unknowns. Thus, the multi-resolution modeling has great potential to be used for real, larger 3-D problems.

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