Thermal Refraction: Impactions for Subglacial Heat Flux

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SUMMARY

Numerical models of glaciers suggest variations in geothermal heat flux influences basal melting. In this study, we demonstrate the effect of thermal refraction on heat flux variations at the glacial-basement interface using a finite difference approximation of the 2D, steady-state, heat flow equation. Thermal refraction occurs as a result of variations in subglacial topography where the thermal conductivity of glacial ice and the solid Earth differ, or in the absence of subglacial topography where contacts between differing conductivity rocks exist. Both models are incompatible with prior topographic-based models of subglacial heat flux. Heat flux can preferentially flow into or around a subglacial valley depending on the thermal conductivity contrast with surrounding rock, with magnitudes at the glacial-basement interface ±20 to 40% of regional geothermal heat flux.

Key words: Thermal Conductivity, Thermal Refraction, Heat Flow, Subglacial Topography

INTRODUCTION

The heat flux at the base of the Antarctic ice sheet is poorly constrained (Larour et al., 2012). Many glacial models use simple geothermal heat flux models to map out the temperature at the base of the Antarctic ice sheet (Figure 1) (Llubes et al., 2006). While more recent geothermal heat flux models derived from seismic tomography and Curie depth estimates are now available (e.g., An et al. 2016; Martos et al. 2017). However, to estimate heat flux, they rely on simple estimates of crustal heat production and thermal conductivity that are very poorly constrained. In this study, we focus on the implications of shallow differences in thermal conductivity on heat flux at the glacial-bedrock interface.

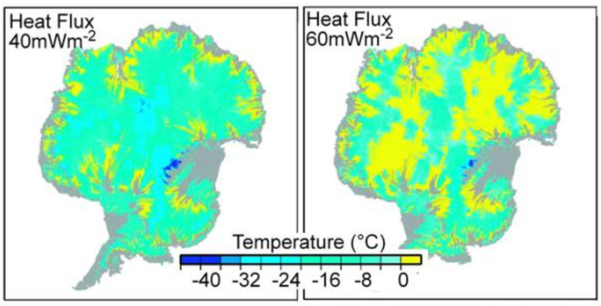

Figure 1. Theoretical models for the base ice temperature based on a modified 1D geotherm (to account for the movement of ice) with a respective heat flux of 40 W m^{-1}K^{-1} and 50 W m^{-1}K^{-1} (Llubes et al., 2006), well within the magnitude of potential thermal refractive effects.

Heat moves from the Earth's mantle through the lithosphere to the surface via the path of least resistance (Beardsmore 2001). Usually this is the shortest path and thus heat will mostly travel vertically due to the difference in temperature between the base and surface of the lithosphere. However, topographic anomalies are known to cause the flow of heat to deviate from a straight path to the surface instead inducing horizontal movement in the flow (Lees 1910, Lachenbruch 1968).

This deviation in heat flux can lead to abnormalities in the heat flux in which certain regions will have lower rates of heat flux compared to others (Veen 2007). This in turn can affect the local thermal gradients, as seen in Figure 2, resulting in rocks beneath the surface being abnormally hot and cold when compared with surrounding rocks.

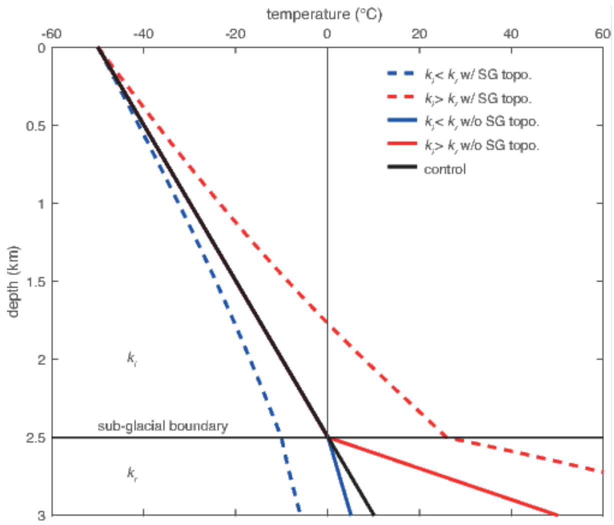

Figure 2. Theoretical thermal gradient through the ice sheet in Figure 4 part A (taken at x = 0 km and presuming k_i = 2.1 W m^{-1} K^{-1}). Blue lines represent an environment where the underlying bedrock is more conductive, red is where the underlying bedrock is less conductive and black represents a gradient in which the two mediums thermal conductivity are identical to act as a control. Solid lines presume heat flux across the subglacial boundary is unaffected by the topography meanwhile the dashed line presume a varied heat flux defined by the thermal refraction method described by this study.

Current methods to describe this deviation of heat flux in subglacial settings use a topographic method (Veen 2007) with solutions that can be demonstrated with upwards continuation (Blackwell 1980). These are based on assumptions about the properties of ice and bed rock which are flawed, and thus the method does not describe the true flow of heat in a subglacial environment. I instead propose a thermal refraction method based on a solution to the steady-state heat equation be used instead, this method is not only accurate but also more versatile, accounting for some local heat flux variations in the absence of subglacial topography.

With these findings we hope to demonstrate how topography affects the flow of heat and the resulting effect on an ice sheets geotherms which alludes to much warmer and cooler sections under the ice sheet as would be suggested by Figure 1. This intern can imply melts in regions not originally theorized by previous studies.

METHOD AND RESULTS

Topographic Method

Originally theorized by Lees (1910) and later expanded by Lachenbruch (1968), this method looks at surface topography (mountains and valleys) and accounts for heat flux into regions of low topography and away from high topography. Veen (2007) uses this method to describe the flow of heat though a subglacial boundary. They proposed that ice is a near perfect insulator and hence does not conduct heat in a similar manner to air on the surface, hence they describe the flow of heat though a subglacial boundary in the same manner as an identical boundary on the surface. However, this method fails to account for thermal conductivity of the ice.

We demonstrate the topographic method by upward continuation of isotherms (Equation 1; Blackwell 1980) which finds the temperature, *T*, at *z* beneath the boundary at location,

$$T(x,z) = T(x,0) + {}^{q}/_{k_r} + \sum_{n=0}^{M} e^{-\frac{2\pi n z}{\lambda}} \left[A_n \cos\left(\frac{2\pi n x}{\lambda}\right) + B_n \cos\left(\frac{2\pi n x}{\lambda}\right)\right] \qquad (1)$$

(q) regional heat flux, (k_r) thermal conductivity of bedrock, (A_n B_n) Fourier coefficients and (λ) length of region of observation. We can find the flow of heat by finding the difference between the temperature at the surface, $T(x,0)$, and immediately beneath the surface, $T(x,dz)$, and multiplying it by the thermal conductivity, k.

Thermal Refraction Method

The thermal refraction solution is derived from the heat equation as described in Thermal Gradients in Continental Crust (Chapman_1986) which describes the following relation,

$$\nabla k \cdot \nabla T + k \nabla^2 T = 0 \qquad (2)$$

We solve the heat equation using a successive relaxation to a finite difference approximation with a cell size of 0.1 km by 0.1 km and the following boundary conditions:

- fixed surface temperature at T_s (x)
- zero flux on the left- and right-hand sides
- fixed heat flux across the base at q_r

Once a temperature solution is found, heat flux at the glacial-basement interface is determined by Fourier's law (i.e. temperature gradient times conductivity). Note, that there is a discontinuity at the boundary thus the flow of heat is determined by averaging the flow of heat just above and below the boundary.

Thermal Conductivity of the Ice

The thermal conductivity of pure ice is approximated by (Paterson 1994),

$$k_i = 2.072 e^{-5.7*10^{-3} T} \qquad (3)$$

That ice at 0°C has a thermal conductivity of ~2.1 W m^{-1} K^{-1} and at -50°C (a common surface temperature in Antarctic glacial environments) ice is found to be even more conductive at 2.8 W m^{-1} K^{-1}.

In practice, the conductivity of glacial ice is lower than the pure ice limit as air bubbles in accumulating snow are trapped as the snow compacts into glacial ice. The lower density of glacial ice results in a lower the thermal conductivity (Figure 3; Paterson 1994). Using density data from Kuivinen (1982) we can estimate the upper and lower bounds of glacial ice thermal conductivity, which can drop to as low as 0.5 W m^{-1} K^{-1} near the surface but rapidly increases with depth to that of pure ice around 300m.

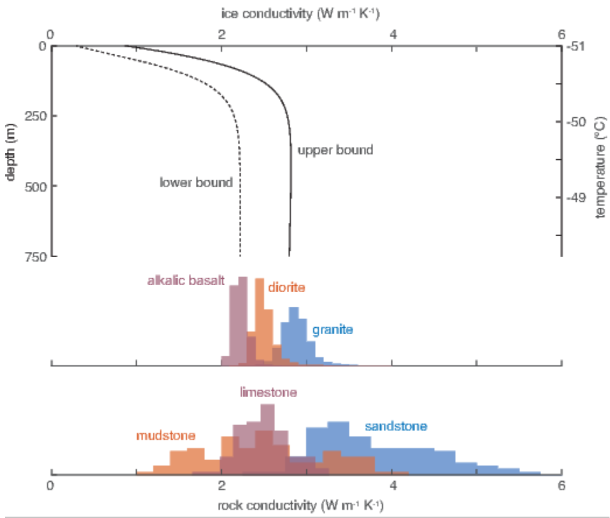

Figure 3. A. Thermal conductivity of ice with respect to depth. The 2 lines show the conductivity of ice when accounting for high porosity near the surface with the dashed line showing the minimum and the solid line representing the maximum. B. Distribution of igneous rock's thermal conductivity calculated using a composition calculation designed by Jennings (2017) on the global igneous rock data base (Gard et al., submitted). C. Distribution of sedimentary rock thermal conductivity from Fuchs_(2013).

Thermal Conductivity of Rock

In Figure 3 we compare the thermal conductivity of ice with that of a few common igneous and sedimentary rocks. Immediately we notice that deep ice not only has a thermal conductivity similar to the average of many common rocks. While rocks can be more conductive than ice (in the case of certain sediments reaching values >5 W m^{-1} K^{-1}) they can also be more resistive (having thermal conductivities as low as 1 W m^{-1} K^{-1}). This immediately puts doubts on the topographic solution claiming that ice is a near perfect insulator when compared to the surrounding bed rock. It also means the refractive effect will switch polarity depending on whether ice has a conductivity lower or greater than that of bedrock (Figure 3).

Model Comparison

To demonstrate the effect of subglacial topography on the heat flux, we construct a simple Gaussian shaped valley in bedrock beneath 1 km of ice with a depth of 1.5 km and a width of 3 km. The thermal conductivity of the bedrock, k_r, is set to 3 W m^{-1} K^{-1} and the ice sheet conductivity, k_i, is 2.1 W m^{-1} K^{-1}. Isotherms (part A) and heat flux across the boundary (part B) for both methods are shown in Figure 4.

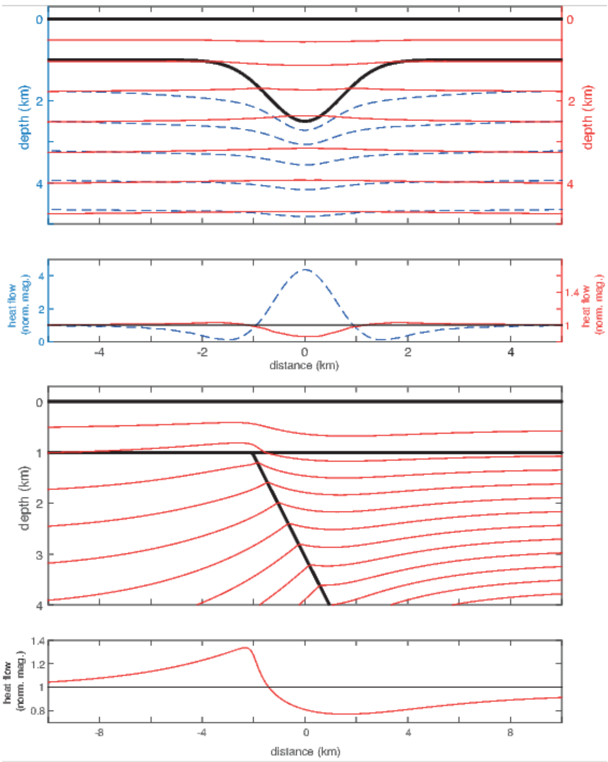

Figure 4. A. Isobars across a theoretical subglacial boundary where the dashed lines are determined with the topographic method (in which k_i=0 W/m^{-1}K^{-1} & k_r=3 W/m^{-1}K^{-1}) and the solid line determined with the reactive method (in which k_i=2.1 W/m^{-1}K^{-1} & k_r=3 W/m^{-1}K^{-1}). Note: The topographic method cannot show the isobars above the boundary. B. Heat flux across the subglacial boundary derived from (part A.) with the topographic and reactive method being represented with the same lines as per part A. C. Isobars across an environment of varying sub-terranean thermal conductivity (with thermal conductivities of 3 W/m^{-1}K^{-1} for the left bedrock slap and 1 W/m^{-1}K^{-1} for the right) but no variation in subglacial topography. D. Heat flux across the subglacial boundary derived from part A. Note how the heat flux is still perturbed even without subglacial topography

As we can see the two methods reveal two completely different results, in both orientation and scale. As expected from the topographic method, heat flows into the subglacial valley as it would on the surface. In contrast, the solution ot the heat equation more accurately shows that heat would refract around the valley due to its low thermal conductivity. The scaling difference between the two methods is likely due to the assumptions of the two methods, i.e., thermal conductivity of ice of ~0 W m^{-1} K^{-1} (infinite thermal resistivity) for the topographic method and 2.1 W m^{-1} K^{-1} for the refraction method. The extreme assumption for the former model is the cause for its greater fluctuations in boundary heat flux.

The orientation and the scaling show how the topographic method is fundamentally flawed as heat would avoid a resistive valley and ice is not a thermally insulator when compared with bedrock. In fact (as will be shown in Figure 5) the topographic solution actually requires the medium above the boundary to transport heat quickly, while air is a good thermal insulator it can very quickly transport heat via convection and this is what gives us the fixed boundary temperature required for the topographic method to be applicable, conditions not found under an ice sheet.

DISCUSSION

Now that have demonstrated the refraction model to describing the flow of heat through subglacial environments, we explore the effects of variations of thermal conductivity contrasts between the ice and bedrock and the potential for thermal refraction in the absence of subglacial topography.

Effects of Varying Thermal Conductivity

Given the range of ice and bedrock thermal conductivities, 0.3 W m^{-1} K^{-1} to 2. W m^{-1} K^{-1} for ice and 1 W m^{-1} K^{-1}- >5 W m^{-1} K^{-1} for bed rock (Figure 3), we construct a set of alternate scenarios for the valley model. By increasing the ratio of thermal conductivities between ice and bed rock the magnitude of the heat flux anomaly increases across the ice-rock boundary (Figure 5).

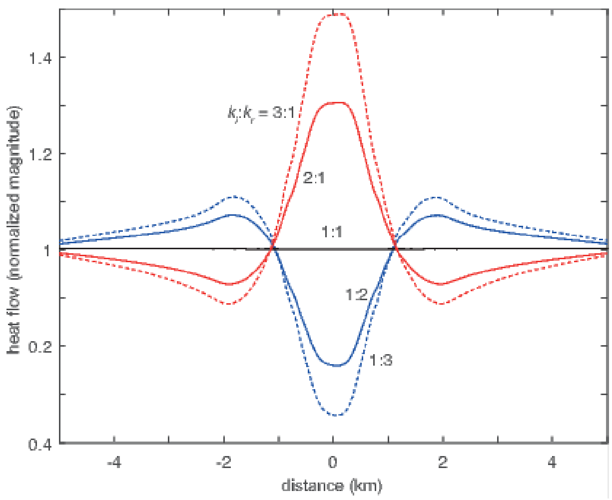

Figure 5. Heat flux across the boundary from Figure 4 part A. with varying conductivity ratios from k_i : k_r = 1:3 - 3:1

In addition, by flipping the ratio and making the ice more conductive, heat flows into the valley rather than around it similar to the topographic solution. We also see in the event of no conductivity difference the heat flux will not vary and instead remain at the regional gradient as it crosses the boundary. When the subglacial geology is felsic plutonics, or rich in quartz containing sediments, heat will likely flow around the valley and into the valley when the bedrock is a highly porous or clay-rich sediments. Mafic plutonics and low porosity basaltic flows may cause very little thermal refraction.

Lack of Topography

The refraction effect also occurs in environments with flat topography (Figure 4C). The topographic model would presume no fluctuations in the flow of heat across the boundary but as part D shows heat will indeed flow though the more conductive medium on the left verses the less conductive medium on the right despite the lack of topography.

Effects on Isotherms

The fundamental difference in the topographic method and this paper's refractive method in mapping boundary heat flux is the assumption that heat either moves into or avoid valleys. These assumptions have an important impact when it comes to the thermal gradient of the ice sheet. In Figure 2 the solid lines show us the thermal gradient of the ice with no topographic effects as we can see the gradient within the ice is not affected by the thermal conductivity of the underlying bed rock. However, when we apply topographic effects (with the refractive model) we immediately notice the thermal gradient within the ice becomes much steeper (in the case of the bed rock being less conductive) and shallower (in the case of the bed rock being more conductive) than its original 1D gradient.

The result is the existence of ice warmer or cooler than what would be implied by a 1D gradient. This can have large ramifications including raising or lowering the strain rate of ice (Goldsby 2001) and causing the ice to melt as seen in Figure 2 with the thermal gradient of a less conductive bedrock exceeding ice's melting point of 0ºC thus implying the ice at the base of the ice sheet has or will melt.

PDE Modelling Solution (Future Plans)

Now that we have demonstrated the effectiveness of the thermal refractive method, we are attempting to improve the method going from my own self-made finite difference method to a premade finite element method. The use of finite elements drastically improves computation speed and accuracy but requires drawing boundaries and giving them either Neuman or Didactic conditions as well as defining the thermal properties of the medium before it can be solved.

We plan to apply this code to improve geothermal heat flux estimates that can be utilized by glacial modelling and improve prediction of basal ice temperatures and melt production.

CONCLUSIONS

In this study, we demonstrate how heat moves though the lithosphere specifically in subglacial environments. The previously used topographic methods of mapping heat flux across a subglacial boundary do not fit solutions to the heat equation where glacial ice has a finite thermal resistivity. In reality, thermal conductivity differences result in refraction of thermal fields, which can occur even in environments with no subglacial topography. Finally, we suggest flow of heat (into and around a subglacial valley) can have large impacts on the thermal gradient resulting in ice melting or remaining solid at depths undefined by a standard 1D thermal gradient. Although this phenomenon is demonstrated for a glacial environments, the effect applies to ice-free environments.

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REFERENCES

An M., Wiens, D., et al., 2015, Temperature, lithosphere–asthenosphere boundary, and heat flux beneath the Antarctic Plate inferred from seismic velocities, J. Geophysics. Res., 120, 8720–8742.

Blackwell, D.D., Steele, J. L., & Brott, C. A., (1980), The terrain effect on terrestrial heat flow, Journal of Geophysical Research, Vol. 85, No.B9, Pages 4757-4772 .

Beardsmore, G. R., & J. P. Cull (2001), Crustal Heat Flow: A Guide to Measurement and Modeling, 324 pp., Cambridge Univ. Press, New York.

Chapman, D. S., (1986), Thermal gradients in the continental crust, Geological Society of London.

Fuchs, S., Schütz, F., Förster, H.-J., Förster, A. (2013): valuation of common mixing models for calculating bulk thermal conductivity of sedimentary rocks: Correction charts and new conversion equations. - Geothermics, 47, 40-52

Gard, M., Hasterok, D., Halpin, J., (submitted 2019), Global whole-rock geochemical database compilation, Earth Systems Science Data

Goldsby, D., & Kohlstedt, D., (2001), Superplastic deformation of ice: Experimental observations, Journal of Geophysical Research, Vol. 106, No.B6, Pages 11,017-11,030

Jennings, S., (2017) A new compositionally-based thermal conductivity model for plutonic rocks, Honors Thesis, Universty of Adelaide

Kuivinen, K. C., et al., (1982), South Pole ice core drilling, 1981-1982. Antarctic Journal of the United States 17(5): 89-91

Lees, C. H. (1910), On the shapes of the isogeotherms under mountain ranges in radio-active districts, Proc. R. Soc. London, Ser. A, 83, 339–346.

Lachenbruch, A. H., (1968), Rapid estimation of the topographic disturbance to supercial thermal gradient, Rev. Geophys., 6 365–400.

Martos, Y., Catalán, M., et al., (2017), Heat flux distribution of Antarctica unveiled, Geophysics. Res. Lett., 44, 11417–11426.

Paterson, W. S. B., (1994), The Physics of Glaciers, Pergamon

Van Der Veen, C.J., et al., (2007), Subglacial topography and geothermal heat flux: Potential interactions with drainage of the greenland ice sheet, Geophys. Res. Lett., 34,

Woschnagg, K., et al., (2002). Temperature profile for glacial ice at the South Pole: Implications for life in a nearby subglacial lake, Proceedings of the National Academy of Sciences of the United States of America, vol. 99, no