The sample boundary effect in the low-frequency measurements of the elastic moduli of rocks

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SUMMARY

In this study we examine the effect of the boundary conditions on the laboratory measurements of the elastic properties of a fluid-saturated sedimentary rock at low frequencies. In laboratory experiments associated with studying fluid effects on elastic properties of a porous rock sample, the tested sample cannot be completely sealed due to the presence of the fluid lines connected to its pore space. These lines form a pore-fluid storage which can affect the results of the elastic moduli measurements of fluid-saturated rocks. We developed a modified version of the Gassmann model which can estimate the bulk moduli of fully saturated rocks in dependence on the capacity of the pore-fluid storage. Here, we compare the predictions of the modified Gassmann model with the moduli measured on an n-decane-saturated limestone sample with the volume of the pore-fluid storage changing from 2 ml to 260 ml. The experimental results were obtained using a low-frequency apparatus based on the forced-oscillation method at a frequency of 0.1 Hz. We demonstrate that the predictions of the modified Gassmann model are in good agreement with the experimental data.

Key words: boundary effect, elastic moduli, Gassmann

INTRODUCTION

Due to a number of advantages associated with the use of low-frequency (LF) laboratory methods for measuring the mechanical properties of rocks (see e.g. Mikhaltsevitch et al., 2014), the LF devices, mainly based on the forced-oscillation approach, have become widespread in many laboratories in recent years (Subramaniyan et al., 2014).

One of the problems that arise when using the LF devices is associated with the correct interpretation of measurement data, inasmuch as the boundary conditions of a tested rock sample can have a crucial impact on the measurement results. Thus, Dunn (1986) and White (1986) demonstrated that the open boundary conditions in the devices using the forced-oscillation method lead to radial fluid motion in a tested rock, thereby causing the dispersion of elastic moduli and attenuation. As was shown by Pimienta et al. (2016), the fluid lines connected to the tested sample can have a significant impact on the results of the moduli measurements in the case of the fully saturated sample, since the line segments located between the sample and the nearest valves cannot be isolated from the sample and form pore-fluid storages, so-called "dead volumes" (Pimienta, 2016), directly connected to the pore space of the sample.

Considering that the Gassmann theory (Gassmann, 1951) is the main instrument commonly used to validate the quality of experimental data obtained on saturated rocks, the purpose of this study is to develop a modified version of the Gassmann model that takes into account the dead volumes and compare the estimates of this model with experimental results. The experiments were carried out on Savonnieres limestone saturated with n-decane at an effective pressure of 7 MPa.

EXPERIMENT

The LF apparatus used in this study is based on the forced-oscillation method and described in detail in Mikhaltsevitch et al. (2014). The mechanical assembly of the apparatus is presented in Figure 1, where the minimum allowable dead volume formed by the segments of the pore-fluid line is highlighted in blue. The apparatus measures the complex Young's modulus and Poisson ratio of rocks in the seismic frequency band at confining pressures of 0 to 50 MPa and at pore pressures of 0 to 20 MPa.

Using the measured Young's modulus E and Poisson ratio ν, the bulk K and shear μ moduli can be computed as

$$K = \frac{E}{3(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \qquad (1)$$

The LF experiments were conducted on a Savonnieres limestone sample with the following physical parameters: the density of the dry sample is 1920 kg/m^3, the porosity is 29%, and the permeability is 450 mD. The diameter of the sample is 38 mm, the length is 71 mm, and the pore space is 23 cm^3. In our experiments, the storage capacity of the pore fluid can be changed from 2 ml to 270 ml. We used this capacity as a variable dead volume to quantify the impact of the dead volume on the measured elastic moduli. The pore fluid used in our measurements was n-decane. The dependence of the elastic moduli on the dead volume was measured with the dead volume gradually increasing from 2 ml to 260 ml at a confining pressure of 10 MPa and a pore pressure of 3 MPa, as well as with the open fluid line at a confining pressure of 7 MPa and an ambient pore pressure. All measurements were carried out at a frequency of 0.1 Hz.

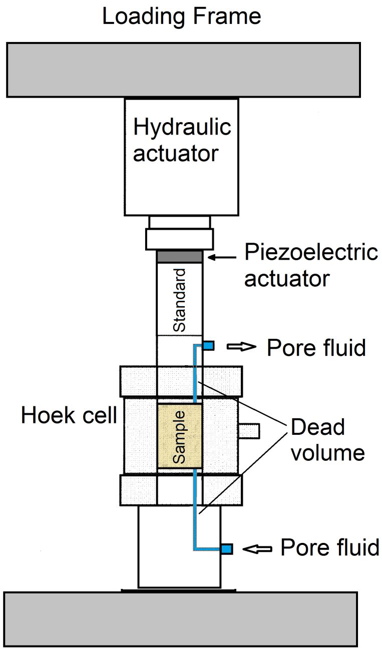

Figure 1. The mechanical assembly of the low-frequency apparatus based on the forced oscillation method. The minimum allowable dead volume formed by the segments of the pore-fluid lines is highlighted in blue.

GASSMANN MODEL

To include the dead volume into the Gassmann model, let us consider a simplified arrangement when the dead volume is surrounded by the rigid non-deformable walls with the exception of the side adjacent to a fluid-saturated isotropic rock sample (Figure 2). We assume that the area of the contact between the sample and the dead volume is negligible compared to the area of the sample surface, so the stress in the sample induced by the applied pressure is homogenous.

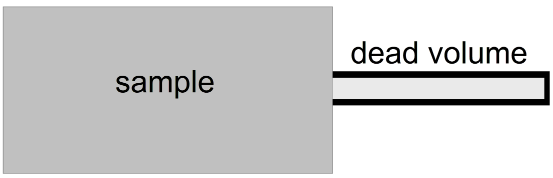

Figure 2. The arrangement of the fluid-saturated sample and the dead volume surrounded by rigid non-deformable walls.

If the total pressure applied to the sample changes by the value ΔP, then this change can be presented as

$$\Delta P = \Delta P_s + \Delta P_f \tag{2}$$

where ΔP_s is the change in pressure borne by the solid frame of the rock and ΔP_f is the change in fluid pressure.

The variation of the fluid pressure changes the volume of both the fluid and the solid frame:

$$\Delta V_f = -(\varphi V + V_D)\Delta P_f / K_f = \varepsilon V \Delta P_f / K_f \tag{3}$$

$$\Delta V_s = \Delta V_{s1} + \Delta V_{s2} \tag{4}$$

where

$$\varepsilon = (\varphi V + V_D)/V = (V_p + V_D)/V \tag{5}$$

$$\Delta V_{s1} = -(1-\varphi)V\Delta P_f / K_s \tag{6}$$

$$\Delta V_{s2} = -V\Delta P_s / K_s \tag{7}$$

In Equations (3) – (7), ΔP_f and ΔV_f are the changes in fluid pressure and in total fluid volume, respectively, V is the volume of the sample, φ is the porosity of the rock, V_D and V_p are the dead and pore volumes, respectively, ΔV_s is the change in the volume of the solid frame caused by fluid pressure (ΔV_{s1}) and by the change in pressure borne by the solid frame (ΔV_{s2}), K_f and K_s are the bulk moduli of the fluid and the solid-frame material, correspondingly. Thus, the relative change of the sample volume can be written as

$$\frac{\Delta V}{V} = \frac{\Delta V_f + \Delta V_s}{V} = -\left(\frac{\varepsilon}{K_f} + \frac{1-\varphi}{K_s}\right)\Delta P_f - \frac{\Delta P_s}{K_s}. \tag{8}$$

The second relation for the relative change of the sample volume can be presented as

$$\frac{\Delta V}{V} = -\frac{1}{K_s}\Delta P_f - \frac{1}{K_d}\Delta P_s \tag{9}$$

where K_d is the drained bulk modulus, $-\frac{1}{K_d}\Delta P_s$ is the fraction of the sample volume change caused by the pressure ΔP_s applied to the evacuated solid frame, $-\frac{1}{K_s}\Delta P_f$ is the fraction of the sample volume change associated with the change in fluid pressure ΔP_f.

The bulk modulus of the fluid-saturated sample is defined as

$$K_{\text{sat}} = -V\Delta P / \Delta V \tag{10}$$

Solving Equations (2), (8) and (9) yields

$$K_{\text{sat}} = K_d + \frac{\left(1 - \frac{K_d}{K_s}\right)^2}{\frac{\varepsilon}{K_f} + \frac{1-\varphi}{K_s} - \frac{K_d}{K_s^2}} \tag{11}$$

Equation (10) is the modified Gassmann equation that takes into account the effect of the fluid-filled dead volume connected with the pore space of the sample. Obviously, in the case when $V_D \rightarrow 0$, we have $\varepsilon \rightarrow \varphi$ and Equation (11) transforms into the standard Gassmann equation.

Let us also note, that the other key element of the Gassmann theory, the independence of the shear modulus from the presence of pore fluid (Berryman, 1999), remains unchanged.

RESULTS

The experimental dependences of the bulk and shear moduli on the dead volume obtained at 0.1 Hz for the fully n-decane saturated Savonnieres sample are presented in Figure 3. As can be seen, the shear modulus is virtually independent of the dead volume, which corresponds to the Gassmann theory, whereas the bulk modulus gradually decreases with increasing the dead volume and approaches the drained modulus.

To compare the measured moduli with the predictions of the standard and modified Gassmann equations, we used the following moduli for the solid-frame material (calcite) and fluid: K_s = 76.8 GPa (Simmons and Wang, 1971) and K_f = 0.86 GPa (White, 1986).

As can be seen in Figure 3, the measured moduli are in good agreement with the estimates computed using Equation (11).

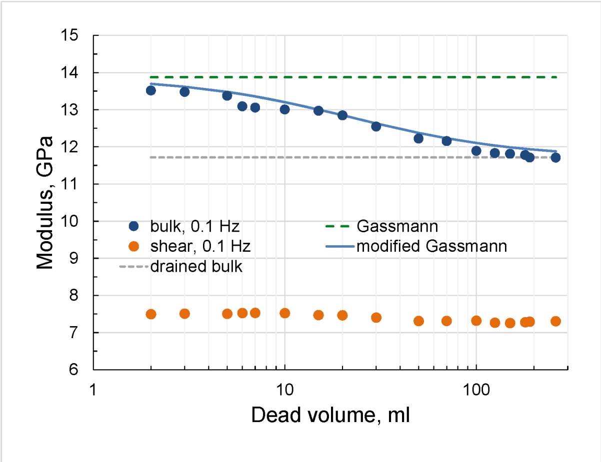

Figure 3. The bulk and shear moduli dependences on the dead volume measured at a frequency of 0.1 Hz and at an effective pressure of 7 MPa. The Gassmann moduli computed using the standard and modified Gassmann equations are shown in the graph with the dashed green and solid blue lines, respectively.

The difference between the bulk moduli computed using the standard and modified Gassmann models is within 3%, if the dead volume does not exceed 25% of the pore space of the sample. Similar, the difference between the estimates based on the modified Gassmann model and the drained bulk moduli is also within 3%, when the dead volume is larger than four volumes of the pore space.

CONCLUSIONS

We investigated the effect of a pore-fluid storage (dead volume) on the low-frequency measurements of the elastic moduli of a fluid-saturated isotropic rock. We developed a modified version of the Gassmann equation that takes into account the effect of the dead volume connected with the pore space of the rock sample. The estimates of the modified Gassmann model were compared with the bulk moduli measured on a Savonnieres limestone sample fully saturated with n-decane. The measurements were carried out using the low-frequency apparatus based on the forced-oscillation method at a frequency of 0.1 Hz under an effective pressure of 7 MPa. The dead volume in our experiments was varying from 2 ml to 260 ml. We showed that the predictions of the modified Gassmann model are in good agreement with the experimental data.

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